The total is 50 pt. Q-n 59 is 10 pt and 2/2/3/3

a. $E(X) = \frac{1}{\lambda} = 1$.

b. $\sigma = \frac{1}{\lambda} = 1$.

c. $P(X \le 4) = 1 - e^{-(1)(4)} = 1 - e^{-4} = .982$.

d. $P(2 \le X \le 5) = (1 - e^{-(1)(5)}) - (1 - e^{-(1)(2)}) = e^{-2} - e^{-5} = .129$.

Q-n 61 is 10 pt and 5/5. In part a), split questions as 1/2/2. In part b), split questions as 2/3

Note that a mean value of 2.725 for the exponential distribution implies $\lambda = \frac{1}{2.725}$. Let X denote the duration of a rainfall event.

a. $P(X \ge 2) = 1 - P(X < 2) = 1 - P(X \le 2) = 1 - F(2; \lambda) = 1 - [1 - e^{-(1/2.725)(2)}] = e^{-2/2.725} = .4800$;
$P(X \le 3) = F(3; \lambda) = 1 - e^{-(1/2.725)(3)} = .6674$;

$P(2 \leqslant X \leqslant 3) = F(3) - F(2) = 0.6674 - 0.5200 = 0.1474$

b. For this exponential distribution, $\sigma = \mu = 2.725$, so $P(X > \mu + 2\sigma) =$
$P(X > 2.725 + 2(2.725)) = P(X > 8.175) = 1 - F(8.175; \lambda) = e^{-(1/2.725)(8.175)} = e^{-3} = .0498$.
On the other hand, $P(X < \mu - \sigma) = P(X < 2.725 - 2.725) = P(X < 0) = 0$, since an exponential random variable is non-negative.

Q-n 37 is 10 pt and 5/5. Split part a) as 2 (sampling distribution of X_bar)/2(for E(X\bar)/1 (for the correct comparison). Use the sample split in pa

37. The joint pmf of X_1 and X_2 is presented below. Each joint probability is calculated using the independence of X_1 and X_2; e.g., $p(25, 25) = P(X_1 = 25) \cdot P(X_2 = 25) = (.2)(.2) = .04$.

	$p(x_1, x_2)$	x_1: 25	40	65	
x_2	25	.04	.10	.06	.2
	40	.10	.25	.15	.5
	65	.06	.15	.09	.3
		.2	.5	.3	

a. For each coordinate in the table above, calculate $\bar{x}$. The six possible resulting $\bar{x}$ values and their corresponding probabilities appear in the accompanying pmf table.

$\bar{x}$	25	32.5	40	45	52.5	65
$p(\bar{x})$	.04	.20	.25	.12	.30	.09

From the table, $E(\bar{X}) = (25)(.04) + 32.5(.20) + ... + 65(.09) = 44.5$. From the original pmf, $\mu = 25(.2) + 40(.5) + 65(.3) = 44.5$. So, $E(\bar{X}) = \mu$.

b. For each coordinate in the joint pmf table above, calculate $s^2 = \frac{1}{2-1}\sum_{i=1}^{2}(x_i - \bar{x})^2$. The four possible resulting s^2 values and their corresponding probabilities appear in the accompanying pmf table.

s^2	0	112.5	312.5	800
$p(s^2)$	.38	.20	.30	.12

From the table, $E(S^2) = 0(.38) + \ldots + 800(.12) = 212.25$. From the original pmf, $\sigma^2 = (25 - 44.5)^2(.2) + (40 - 44.5)^2(.5) + (65 - 44.5)^2(.3) = 212.25$. So, $E(S^2) = \sigma^2$.

Q-n 46 is 3/3/4. In part a), split as 1/2 for two questions there. The same should be done for part b.
In part c), take off 1 pt if there is no explanation.

46. $\mu = 70$ GPa, $\sigma = 1.6$ GPa

a. The sampling distribution of $\bar{X}$ is centered at $E(\bar{X}) = \mu = 70$ GPa, and the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{1.6}{\sqrt{16}} = 0.4$ GPa.

b. With $n = 64$, the sampling distribution of $\bar{X}$ is still centered at $E(\bar{X}) = \mu = 70$ GPa, but the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{1.6}{\sqrt{64}} = 0.2$ GPa.

c. $\bar{X}$ is more likely to be within 1 GPa of the mean (70 GPa) with the second, larger, sample. This is due to the decreased variability of $\bar{X}$ that comes with a larger sample size.

Q-n 47 is 10 pt and 5/5

47.

a. In the previous exercise, we found $E(\bar{X}) = 70$ and $SD(\bar{X}) = 0.4$ when $n = 16$. If the diameter distribution is normal, then $\bar{X}$ is also normal, so
$P(69 \le \bar{X} \le 71) = P\left(\frac{69-70}{0.4} \le Z \le \frac{71-70}{0.4}\right) = P(-2.5 \le Z \le 2.5) = \Phi(2.5) - \Phi(-2.5) = .9938 - .0062 =$ .9876.

b. With $n = 25$, $E(\bar{X}) = 70$ but $SD(\bar{X}) = \frac{1.6}{\sqrt{25}} = 0.32$ GPa. So, $P(\bar{X} > 71) = P\left(Z > \frac{71-70}{0.32}\right) =$ $1 - \Phi(3.125) = 1 - .9991 = .0009$.